wall $\Sigma = \{|z| = R_0\}$ [3]

$$\frac{\partial h}{\partial t} + \mathrm{div}_\Sigma \left\{ \frac{\sigma_*(h)\, h^3}{3\mu} \nabla_\Sigma \left(\Delta_\Sigma h + \frac{h}{R_0^2} \right) + \frac{h^2}{2\mu} \nabla_\Sigma \sigma_*(h) \right\} = 0.$$

The dependence $\sigma_*(h) = \sigma((1 - \beta h)\theta_0 + \beta h \theta_\infty)$ is obtained here as a result of asymptotic integration of the heat conduction equation for $h/R_0 \to 0$. The linearization is carried out in the constant layer thickness $h = R - R_0$.

In conclusion we note that the critical thermocapillary numbers, making the operator $\hat{L}_k$ vanish, were obtained in [4]. The branching of stationary solutions of the complete equations of thermocapillary convection was established in [5] near the critical Marangoni numbers. These numbers were calculated in [6] for a nondeformed free boundary.

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THEORETICAL AND EXPERIMENTAL STUDY OF CONVECTION IN A LIQUID LAYER WITH LOCAL HEATING

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The existence of shearing forces associated with surface tension at phase boundaries (liquid–liquid, liquid–gas) may have a significant effect on heat and mass transfer in a liquid. In the case where a temperature gradient is created in the volume of liquid being studied, surface thermocapillary forces – due to their low inertia – may lead to the development of fast-moving hydrodynamic flows [1, 2]. These effects become particularly important in space technology in connection with the study of the behavior of materials (melts) under low-gravity conditions, when the role of thermogravitational convection becomes negligibly small [3]. Possible applications here include crystal growth, welding, and the production of foamed materials in space.

The phenomenon of thermocapillary convection (TCC) (Marangoni effect) makes some contribution to mass transfer in normal production processes as well. In the laser treatment of the surface of metals, TCC may play an important role in the alloying and nitriding of different grades of steel [4]. With allowance for the change in the form of the surface under the influence of TCC, possible uses of TCC include the production of diffraction gratings [5] and a new type of photographic process called thermoextensography [6]. This

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TABLE 1

Substance	σ, dyn/cm^2	ρ, g/cm^3	η, g/(cm·sec)	c_p, cal/(g·deg)	λ, cal/(cm·sec·deg)	β, deg^{-1}	Ma	Gr	ΔT	Pr
Alcohol	22,3	0,79	$1{,}2\cdot10^{-2}$	0,59	$4{,}2\cdot10^{-4}$	$110\cdot10^{-5}$	$5{,}4\cdot10^{5}$	$7{,}0\cdot10^{6}$	10	16
Kerosene	27	0,82	$1{,}49\cdot10^{-2}$	0,48	$3{,}6\cdot10^{-4}$	$90\cdot10^{-5}$	$4{,}7\cdot10^{5}$	$2{,}2\cdot10^{6}$	10	28
Acetone	23,7	0,79	$0{,}32\cdot10^{-2}$	0,51	$3{,}8\cdot10^{-4}$	$143\cdot10^{-5}$	$2{,}2\cdot10^{6}$	$1{,}05\cdot10^{8}$	10	4,1

process is based on the action of the Marangoni effect during the exposure of different materials to laser radiation. In biotechnology processes, TCC may also become the basis for the synthesis of new types of products [7]. In connection with these potential applications of TCC, it would be of considerable interest to augment the results in [1, 2] and perform a detailed comparison of experimental data and numerical results.

Formidable problems are encountered in experimental attempts to determine the contribution of interfacial forces to mass transfer in a liquid, since thermocapillary convection needs to be modeled in thin layers (H < 0.5 cm) with a sufficiently large free surface. The use of various types of visualization methods – such as the introduction of tracers, dyes, or particles – may together with the heat source seriously disturb the test conditions and result in sizable errors [8]. The method of photochromic visualization, involving impulsive heating of a medium by laser radiation with the simultaneous appearance of a colored line, makes it possible to avoid these problems.

Numerical modeling was also performed to more fully reveal the heat- and mass-transfer characteristics seen in the local heating of a liquid by pulsed laser radiation. All of the constants in the calculations were chosen so as to be as close as possible to the experimental data and, in contrast to [2], we considered heat removal from the surface. With agreement of the experimental and theoretical curves, it turned out to be possible to reliably determine the temperature at the point of interaction of the radiation with the liquid surface.

We conducted studies for three types of liquid – ethyl alcohol, kerosene, and acetone. Table 1 shows the main parameters of these liquids. Each liquid was in the form of a solution with molecules of FKhV uniformly distributed over the volume. The laser radiation produced a change in the orientation of the FKhV molecules with a simultaneous change in the optical properties (absorption spectrum). There was no change in concentration. Thus, the motion of the liquid was due to thermocapillary convection, and capillary-concentrative convection was absent. A change in the concentration of FKhV in the solution was accompanied by a change in the amount of energy absorbed.

We are examining the convective motion of an incompressible fluid filling a circular cylindrical cuvette of radius R = 5 cm when a short pulse $\tau = 10^{-7}$ sec of ultraviolet radiation, acting as a source of heat in the liquid, is passed through the center of the cuvette. In the dimensionless variables used in [2], the system of Navier–Stokes and heat-conduction equations that describes the process has the form

$$\frac{\partial\omega}{\partial t} + u\frac{\partial\omega}{\partial r} + v\frac{\partial\omega}{\partial z} + \frac{\omega u}{r} = \mathrm{Pr}\left(\nabla^2\omega - \frac{\omega}{r^2}\right) + \mathrm{Gr}\,\mathrm{Pr}^2\frac{\partial\Theta}{\partial r}; \tag{1}$$

$$\frac{\partial\Theta}{\partial t} + u\frac{\partial\Theta}{\partial r} + v\frac{\partial\Theta}{\partial z} = \nabla^2\Theta; \tag{2}$$

$$\nabla^2\psi - \frac{2}{r}\frac{\partial\psi}{\partial r} = r\omega, \quad \omega = \frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}, \tag{3}$$

$$u = \frac{1}{r}\frac{\partial\psi}{\partial z}, \quad v = -\frac{1}{r}\frac{\partial\psi}{\partial r}, \quad \nabla^2 \equiv \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}.$$

Conditions expressing the balance of the viscous and thermocapillary forces (Marangoni effect) and heat removal are assigned on the free surface of the liquid (z = h)

$$\partial u/\partial z = \mathrm{Ma}\,\partial\Theta/\partial r, \ \partial\Theta/\partial z = -\gamma\Theta.$$

The boundary conditions on the lateral walls (r = 1) and the bottom (z = 0) of the cuvette correspond to adhesion of the liquid $\psi = 0$, $\partial\psi/\partial n = 0$. The below symmetry conditions are

assigned on the symmetry axis

$$\psi(r = 0, z, t) = \omega(r = 0, z, t) = \partial\Theta(r = 0, z, t)/\partial r = 0,$$

while the initial conditions are the conditions of stationariness of the liquid

$$\psi(r, z, t = 0) = \omega(r, z, t = 0) = 0$$

and the assigned axisymmetric temperature distribution

$$\Theta(z, t = 0) = \exp(\alpha(z - h)), \quad 0 \leqslant r \leqslant a,$$
$$\Theta(z, t = 0) = 0, \qquad r > a.$$

Problem (1)-(3) includes several dimensionless parameters: the Marangoni number Ma = $-\frac{\partial\sigma}{\partial T}\frac{R(T_1 - T_0)}{\rho\nu\varkappa}$, the Grashof number Gr = $g\beta R^3(T_1 - T_0)/\nu^2$, the Prandtl number Pr = ν/a, the heat-removal coefficient $\gamma = \gamma' R/\varkappa c_p\rho$, and the absorption coefficient α. The latter is not known from the experimental data and is determined as follows. The store of heat in the system

$$Q = c_p\rho\int(T_f - T_0)dV = (T_1 - T_0)c_p\rho\pi Ra^2(1 - e^{-\alpha H/R})/\alpha' \qquad (4)$$

is equal to the energy of the laser pulse E = 0.05 J. Energy losses are not considered. The unknowns in Eq. (4) are the quantity α and the maximum temperature in the heated liquid T_1.

At $\alpha \to 0$ and the depth H = 0.5 cm, the energy of the pulse is sufficient to uniformly heat a column of liquid of the radius a' = 0.055 cm to T_1 = 297 K (i.e., heating $\Delta T = T_1 - T_0$ = 4 K, where T_0 is the temperature of the unheated liquid, equal to 293 K). By increasing T_1, we can use (1.4) to obtain the value of α for performing the calculations.

We used the experimental unit described in [1] and the method of calculation developed in [2].

Figure 1 shows the trajectory of a liquid particle of ethyl alcohol from the center to the surface of the layer (the empirically-observed change in the radius of the colored spot over time). The solid lines are experimental curves, while the dashed lines are theoretical curves. Curves 1 and 2 correspond to layer depths of 0.1 and 0.4 cm, a pulse energy E = 0.05 J, an FKhV concentration C = 0.2 g/liter, Ma = $5.4\cdot10^5$, and Gr = $5.8\cdot10^6$. The value of γ was chosen as follows. According to [11], the coefficient of heat removal at a gas–liquid boundary for a quiescent medium $\gamma' = 1.4\cdot10^{-3}$ cal/(cm^2·sec·deg). Insertion of this value into the numerical calculations did not produce a significant slowing of the external boundary of the colored spot, which in turn caused the theoretical curves to deviate appreciably from the experimental curves. In light of this, in the calculations we gradually increased γ to the value $\gamma' = 7.0\cdot10^{-3}$ cal/(cm^2·sec·deg). This corresponded to the increase in heat transfer from the surface with allowance for the motion of the liquid [12] and resulted in minimal differences between the curves. In comparing the results, we found that good agreement between the theory and experiment was obtained when the temperature gradient in the liquid ΔT = 10 K, i.e., when the maximum temperature seen on the surface of the liquid T_1 = 303 K and α = 46.

Figure 2 shows the dependence of the radius of the colored spot at the moment of time t = 1 sec on the height of the liquid column for kerosene. As could be seen from Fig. 1, convective motion of the liquid dies out by this moment of time, while the radius of the colored spot is close to the limiting value. The solid curve shows experimental results, while the dashed curve shows theoretical results. Values of the parameters for kerosene are shown in the table. With an increase in the depth of the liquid layer, the bottom of the cuvette ceases to have an effect on the radius of motion near the surface. Beginning with 0.4 cm, the radius of the colored spot was nearly independent of the thickness of the liquid layer and was determined only by interfacial and gravitational forces. Figure 2 shows the good agreement between the experimental and theoretical results. Since the dependences of the radius of the spot on height H for alcohol at t = 1 sec were presented independently in [1] and [2], they are not presented here. However, their comparison showed that the difference in the behavior of the theoretical and experimental curves is negligible.

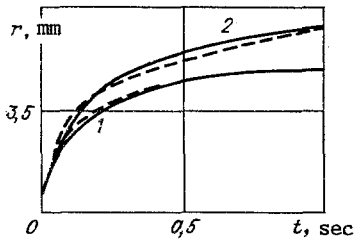

Fig. 1

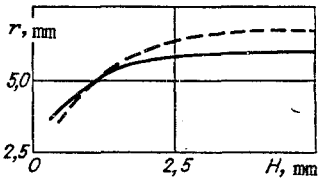

Fig. 2

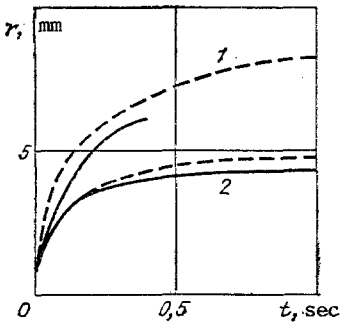

Fig. 3

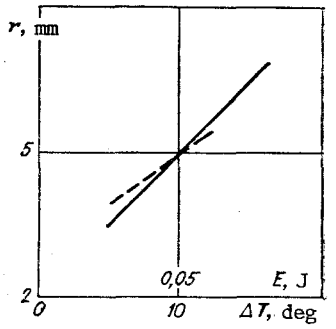

Fig. 4

Figure 3 shows the change in the radius of the colored spot with time for acetone and kerosene (curves 1 and 2). It should be noted that it was difficult to measure the radius r at t > 0.4 sec in the experiments with acetone due to its intensive vaporization. Severe curvature of the circumference of the colored spot was seen on the free surface, this curvature having led to large errors in the measurements.

It could be seen from special experiments conducted with a concentration C = 0.2 g/liter that the absorption coefficient for all three liquids is determined by the quantity of FKhV molecules present per unit volume of the solvent, not by the properties of the test liquid itself. With a decrease in the concentration of FKhV, the absorption coefficient became dependent on the properties of the solvent. The experimental results are shown by the solid curves, while the theoretical results are shown by the dashed curves. It is apparent that the difference between theory and experiment is greatest in the case of acetone, it being larger due to the fact that evaporation from the liquid surface was not considered in the calculations.

A change in the energy of the laser pulse acting on the medium led to a corresponding change in T_1 in the liquid. As a result, there was also a change in the radius of the colored spot. Since satisfactory agreement was obtained between the sets of results with E = 0.05 J and a temperature drop ΔT = 10 K, it is interesting to examine the motion of the liquid at other temperatures (energies). The solid line in Fig. 4 shows the change in the radius of the spot r in relation to the energy of the pulse at the moment of time t = 1 sec (E was changed within the range from 0.025 to 0.08 J with a FKhV concentration C = 0.2 g/liter). In the theoretical model being used here, the change in pulse energy corresponds to a change in the heating of the liquid ΔT. Using the value for the absorption coefficient α = 46, calculations were performed for a temperature drop ΔT from 5 to 12 K. It follows from a comparison of the data obtained for the slope tangents of the lines in the indicated range of the parameters $E(\Delta T)$ that the deviation is no greater than 25%.

It would be incorrect to speak of an increase in the difference between theory and experiment with a change in E away from 0.05 J, since here we are comparing only the slope tangents of the curves and assuming that the relationship between E and T is linear.

Thus, the above analysis of experimental results has shown that intensive heat and mass transfer is seen in liquid under the influence of thermocapillary forces, while the good agreement between the results confirms the correctness of the chosen theoretical model.

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EVOLUTIONARY EQUATION FOR PERTURBATIONS IN A TWO-LAYER FILM FLOW

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We will examine the simultaneous motion of two films of immiscible liquids flowing under the influence of gravity. Such flows are encountered in certain types of extraction columns. The chosen coordinate system is shown in Fig. 1. The film bounded by the solid wall will henceforth be designated as the first film, while the film having the free boundary will be referred to as the second film. The quantities pertaining to these films will be denoted by the subscripts 1 and 2, respectively.

The equations which describe the motion of such a system permit a solution to be obtained with plane phase and free boundaries, regardless of the rates of flow of the liquids. Here, the profiles of longitudinal velocity are equal to

$$U_{10} = \frac{g}{2\nu_1}\left[2\left(H_{10} + H_{20}\rho_2/\rho_1\right)y - y^2\right],$$

$$U_{20} = \frac{g}{2\nu_2}\left[2H_{10}H_{20}\left(\mu_2/\mu_1 - 1\right) + H_{10}^2\left(\nu_2/\nu_1 - 1\right) + 2\left(H_{10}+H_{20}\right)y - y^2\right]. \quad (1)$$

Here, ν_i and μ_i are the kinematic and absolute viscosities; ρ_i is density; H_{i0} is the thickness of the liquid film.

However, even with low flow rates, the flow (1) may become wavelike due to instability. Using as scales characteristic values of the quantities pertaining to the first film – especially the thickness H_{10} and the mean-flow-rate velocity U_0 – for nonwavy flow with the

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